# Math 181: Problem Set #2

Jesse Kass

Winter 2024

Due in 1 week at the start of class.

## Problem 1

The 1855 Catalogue for Furman University is available online. Find it and look up who was the math professor at time. Write down his name as your answer to this question.

## Problem 2

Find an electronic copy of one of the mathematics textbooks listed in the 1855 Catalogue for Furman University. Then tell me (a) which book you selected and (b) how someone can find the book.

## Problem 3

Figure 1 displays a Calculus rule for computing derivatives from *Elements of the Differential and Integral Calculus* by Elias Loomis, a textbook that T. E. Hart taught from. Find the rule in a modern calculus textbook (for example, the textbook assigned for Math 19B/Math 20B). Explain what modern textbook you looked and reproduce its description of the rule. List one difference and one similarity between Loomis's textbook and the modern textbook.

## Problem 4

Figure 2 shows a theorem from Elias Loomis's textbook *Elements of Geometry and Conic Sections*. Read the theorem and its proof. The proof references earlier results in the textbook. Using a word processor (Word or TeX or...), type up your own self-contained version of the theorem and proof that does not reference earlier parts of the Loomis textbook using more modern, accessible language (as though it was a homework solution).

What changes did you make to the text? Did you find an errors or unclear points in Loomis's argument?

**24.** *The differential of the product of two functions dependent on the same variable is equal to the sum of the products obtained by multiplying each function by the differential of the other.*

Let us designate two functions by $u$ and $v$, and suppose them to depend on the same variable $x$; then, when $x$ is increased so as to become $x+h$, the new functions, Art. 23, may be written

$$u' = u + \mathrm{A}h + \mathrm{B}h^2,$$
$$v' = v + \mathrm{A}'h + \mathrm{B}'h^2.$$

If we multiply together the corresponding members of these equations, we shall have

$$\begin{aligned} u'v' = uv &+ \mathrm{A}vh + \mathrm{B}vh^2 \\ &+ \mathrm{A}'uh + \mathrm{A}\mathrm{A}'h^2 + \text{ etc.,} \\ &\qquad\qquad + \mathrm{B}'uh^2 + \text{ etc.,} \end{aligned}$$

where, it will be observed, the terms omitted contain powers of the increment higher than $h^2$.

Transposing, and dividing by $h$, we have

$$\frac{u'v' - uv}{h} = \mathrm{A}v + \mathrm{A}'u + \text{other terms containing } h.$$

When we make $h$ equal to zero, Art. 15, the terms containing $h$ disappear, and we have

$$\frac{d(uv)}{dx} = \mathrm{A}v + \mathrm{A}'u,$$

or, multiplying by $dx$,

$$d(uv) = v\mathrm{A}dx + u\mathrm{A}'dx.$$

But $\quad \mathrm{A}dx$ is equal to $du$,

and $\quad \mathrm{A}'dx$ is equal to $dv$;

hence $$d(uv) = vdu + udv, \qquad (1)$$

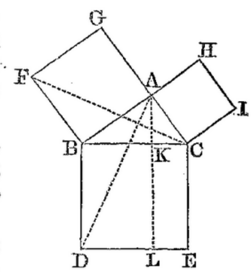
which was the proposition to be demonstrated.

Figure 1: The product rule in Loomis's *Elements of the Differential and Integral Calculus*

PROPOSITION XI. THEOREM.

*In any right-angled triangle, the square described on the hypothenuse is equivalent to the sum of the squares on the other two sides.*

Let ABC be a right-angled triangle, having the right angle BAC; the square described upon the side BC is equivalent to the sum of the squares upon BA, AC.




On BC describe the square BCED, and on BA, AC the squares BG, CH; and through A draw AL parallel to BD, and join AD, FC.

Then, because each of the angles BAC, BAG is a right angle, CA is in the same straight line with AG (Prop. III., B. I.). For the same reason, BA and AH are in the same straight line.

The angle ABD is composed of the angle ABC and the right angle CBD. The angle FBC is composed of the same angle ABC and the right angle ABF; therefore the whole angle ABD is equal to the angle FBC. But AB is equal to BF, being sides of the same square; and BD is equal to BC for the same reason; therefore the triangles ABD, FBC have two sides and the included angle equal; they are therefore equal (Prop. VI., B. I.).

But the rectangle BDLK is double of the triangle ABD, because they have the same base, BD, and the same altitude, BK (Prop. II., Cor. 1); and the square AF is double of the triangle FBC, for they have the same base, BF, and the same altitude, AB. Now the doubles of equals are equal to one another (Axiom 6, B. I.); therefore the rectangle BDLK is equivalent to the square AF.

In the same manner, it may be demonstrated that the rectangle CELK is equivalent to the square AI; therefore the whole square BCED, described on the hypothenuse, is equivalent to the two squares ABFG, ACIH, described on the two other sides; that is,

$$BC^2 = AB^2 + AC^2.$$

Cor. 1. The square of one of the sides of a right-angled

Figure 2: A proof from*Elements of Geometry and Conic Sections.*

You may want to look at earlier parts of the Loomis textbook. A link to the book is on the course website.

## Collaboration Policy

With each week's homework, you must turn in a one paragraph description of all the resources you used on that homework. You must mention any person you talked to about the problems, any book you looked at, any online resource (Wikipedia, Chegg,...) that you used. A sample paragraph is

> On this week's homework, I worked on the problem set collaboratively with Gauss and Grothendieck at The Redroom during happy hour. We found an Alex Jones video (http://youtube.blah.com) that gave a really

clear explanation of Fermat's Last Theorem. We got really stuck on Problem 5, and so we went to Chegg.com and paid an online tutor ("Zariski") $50 to solve the problem for us. He said the problem was too hard for him. So I logged into my TruthSocial account (@CobraTatesThesis) and posted the question with @realDonaldTrump tagged. He responded with a tremendous, really fantastic solution to the problem, which by the way, Biden can't solve. At this point, it was midnight and I still had four more problems to go, so I just gave the questions to ChatGPT and cut-and-pasted the answers.