Math 181: Problem Set #3

Jesse Kass

Winter 2024

Due in 1 week at the start of class.

Make sure you have read the first three chapters of Stedall's *The History of Mathematics: A Very Short Introduction* and sections 14.1–14.2 of Katz's *A History of Mathematics*.

Problem 1

Look up the dates of the following events: (a) the birth of René Descartes, (b) the publication of Descartes's book *La Géométrie*, (c) the death of Descartes, (d) the publication date of *Élémens d'algèbre*, by Louis Pierre Marie Bourdon, (e) the publication of *Elements of Algebra on the Basis of M. Bourdon Embracing Sturm's and Horner's Theorems and Practical Examples* by Charles Davies, and (f) the founding of the Military Academy at West Point.

Then plot all the dates on a timeline.

Problem 2

Figure is a selection from Descartes' *La Géométrie*. Make your own translation using a dictionary, Google Translate, whatever you want.

Problem 3

The statement of the Rule of Signs in Figure 2 is from the textbook *First Course in Theory of Equations*. Displayed in Figure 3 is an accompanying series of exercises. Solve Exercise 4.

Problem 4

Figures 4 and 5 describe a selection from Descartes' *La Géométrie* explaining how to multiply geometrically. He illustrates the idea with Figure 6.

Verify his claim by showing the length of the segment BE equals the product of the lengths of BD and BC.

372 LA GEOMETRIE.

Combien il peut y auoir de racines en chaſq; Equatiõ.

Scachés donc qu'en chaſque Equation, autant que la quantité inconnue a de dimenſions, autant peut il y auoir de diuerſes racines, c'eſt a dire de valeurs de cete quantité. car par exemple ſi on ſuppoſe x eſgale a 2; oubien $x - 2$ eſgal a rien; & derechef $x \infty 3$; oubien $x - 3 \infty 0$; en multipliant ces deux equatiòns $x - 2 \infty 0$, & $x - 3 \infty 0$, l'vne par l'autre, on aura $xx - 5x + 6 \infty 0$, oubien $xx \infty 5x - 6$, qui eſt vne Equation en laquelle la quantité x vaut 2 & tout enſemble vaut 3. Que ſi derechef on fait $x - 4 \infty 0$, & qu'on multiplie cete ſomme par $xx - 5x + 6 \infty 0$, on aura $x^3 - 9xx + 26x - 24 \infty 0$, qui eſt vne autre Equation en laquelle x ayant trois dimenſions a auſſy trois valeurs, qui ſont 2, 3, & 4.

Quelles ſont les fauſſes racines.

Mais ſouuent il arriue, que quelques vnes de ces racines ſont fauſſes, ou moindres que rien. comme ſi on ſuppoſe que x deſigne auſſy le defaut d'vne quantité, qui ſoit 5, on a $x + 5 \infty 0$, qui eſtant multipliée par $x^3 - 9xx + 26x - 24 \infty 0$ fait

$$x^4 - 4x^3 - 19xx + 106x - 120 \infty 0$$

pour vne equation en laquelle il y a quatre racines, a ſçauoir trois vrayes qui ſont 2, 3, 4, & vne fauſſe qui eſt 5.

Figure 1: selection from Descartes' *La Géométrie*

72 *ISOLATION OF REAL ROOTS* [CH. VI

DESCARTES' RULE. ***The number of positive real roots of an equation with real coefficients is either equal to the number of its variations of sign or is less than that number by a positive even integer. A root of multiplicity m is here counted as m roots.***

For example, $x^6-3x^2+x+1=0$ has either two or no positive roots, the exact number not being found. But $3x^3-x-1=0$ has exactly one positive root, which is a simple root.

Figure 2: Dickson's formulation of the Rule of Signs

EXERCISES

Prove by Descartes' rule the statements in Exs. 1–8, 12, 15.

1. An equation all of whose coefficients are of like sign has no positive root. Why is this self-evident?

2. There is no negative root of an equation, like $x^5-2x^4-3x^2+7x-5=0$, in which the coefficients of the odd powers of x are of like sign, and the coefficients of the even powers (including the constant term) are of the opposite sign. Verify by taking $x=-p$, where p is positive.

3. $x^3+a^2x+b^2=0$ has two imaginary roots if $b\neq 0$.

4. For n even, $x^n-1=0$ has only two real roots.

5. For n odd, $x^n-1=0$ has only one real root.

6. For n even, $x^n+1=0$ has no real root; for n odd, only one.

7. $x^4+12x^2+5x-9=0$ has just two imaginary roots.

8. $x^4+a^2x^2+b^2x-c^2=0$ ($c\neq 0$) has just two imaginary roots.

Figure 3: Exercises from Dickson

BOOK I

Problems the Construction of Which Requires Only Straight Lines and Circles

ANY problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction.[1] Just as arithmetic consists of only four or five operations, namely, addition, subtraction, multiplication, division and the extraction of roots, which may be considered a kind of division, so in geometry, to find required lines it is merely necessary to add or subtract other lines; or else, taking one line which I shall call unity in order to relate it as closely as possible to numbers,[2] and which can in general be chosen arbitrarily, and having given two other lines, to find a fourth line which shall be to one of the given lines as the other is to unity (which is the same as multiplication); or, again, to find a fourth line which is to one of the given lines as unity is to the other (which is equivalent to division); or, finally, to find one, two, or several mean proportionals between unity and some other line (which is the same

Figure 4: A selection from Descartes' *La Géométrie*

as extracting the square root, cube root, etc., of the given line.[3] And I shall not hesitate to introduce these arithmetical terms into geometry, for the sake of greater clearness.

For example, let AB be taken as unity, and let it be required to multiply BD by BC. I have only to join the points A and C, and draw DE parallel to CA; then BE is the product of BD and BC.

Figure 5: A selection from Descartes' *La Géométrie*, continued

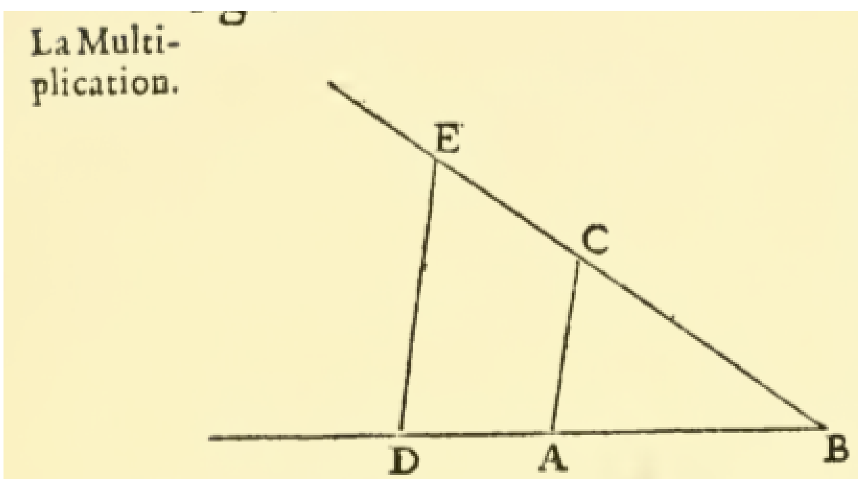

Figure 6: A diagram from Descartes' *La Géométrie*

Collaboration Policy

With each week's homework, you must turn in a one paragraph description of all the resources you used on that homework. You must mention any person you talked to about the problems, any book you looked at, any online resource (Wikipedia, Chegg,...) that you used. A sample paragraph is

> On this week's homework, I worked on the problem set collaboratively with Gauss and Grothendieck at The Redroom during happy hour. We found an Alex Jones video (http://youtube.blah.com) that gave a really clear explanation of Fermat's Last Theorem. We got really stuck on Problem 5, and so we went to Chegg.com and paid an online tutor ("Zariski") $50 to solve the problem for us. He said the problem was too hard for him. So I logged into my TruthSocial account (@CobraTatesThesis) and posted the question with @realDonaldTrump tagged. He responded with a tremendous, really fantastic solution to the problem, which by the way, Biden can't solve. At this point, it was midnight and I still had four more problems to go, so I just gave the questions to ChatGPT and cut-and-pasted the answers.