Math 201: Problem Set #2

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Winter 2024

Due in 1 week.

Problem 1

Let b be a vector space over a field k with a basis $\{v_1, v_2\}$, and $\alpha \colon V \to V$ be the map defined by $\alpha(\lambda_1 v_1 + \lambda_2 v_2) = \lambda_2 v_1 + \lambda_2 v_2$.

1. Prove that α is a k-module homomorphism.
2. Make V into a $k[x]$-module by defining $x \cdot v = \alpha(v)$ (i.e. the construction described in the textbook in the section "Example: ($F[x]$-module)." Describe all the $k[x]$-submodules of V.
3. Are the $k[x]$-submodules of V the same as the k-submodules? Justify your answer.

Problem 2

1. Let $R = \mathbb{Z}$. Is R isomorphic to $2 \cdot R$ as R-modules? Justify your answer.
2. Let $R = \mathbb{Z}/6$. Is R isomorphic to $2 \cdot R$ as R-modules? Justify your answer.

Problem 3

Solve Exercise 8 in section 10.2 of Dummit and Foote.

Problem 4

Solve Exercise 10 in section 10.2 of Dummit and Foote.

Collaboration Policy

With each week's homework, you must turn in a one paragraph description of all the resources you used on that homework. You must mention any person you talked to about the problems, any book you looked at, any online resource (Wikipedia, Chegg,...) that you used. A sample paragraph is

> On this week's homework, I worked on the problem set collaboratively with Gauss and Grothendieck at The Redroom during happy hour. We found an Alex Jones video (http://youtube.blah.com) that gave a really clear explanation of Fermat's Last Theorem. We compared our solutions against a solution key that we found on the /commutativealgebra/ board of 4chan (http://blah.blah.edu). We also got really stuck on Problem 5, and so we went to Chegg.com and paid an online tutor ("Zariski") $50 to solve the problem for us.