Math 201: Problem Set #3

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Due in 1 week.

Problem 1

Let $V = (\mathbb{Z}/2)^3$ be the standard three dimensional vector space over the field $\mathbb{Z}/2$ with two elements. Define $\alpha \colon V \to V$ by

$$\alpha(x, y, z) = (x + y, y + z, x + z).$$

1. Prove that α is a k-module homomorphism.
2. Make V into a $k[x]$-module by defining $x \cdot v = \alpha(v)$ (i.e. use the construction described in the textbook in the section "Example: ($F[x]$-module)." Is V a finitely generated $k[x]$-module? Prove that your answer is correct.
3. As a module over $k[x]$, is V a torsion module?
4. As a module over k, is V a torsion module?

Problem 2

Solve Exercise 12 in section 10.3 of Dummit and Foote.

Problem 3

Solve Exercise 16 in section 10.3 of Dummit and Foote.

Problem 4

Solve Exercise 17 in section 10.3 of Dummit and Foote.

(Can you figure out what happens with the Chinese Remainder Theorem when $R = M = \mathbb{Z}$ but you allow for an infinite number of ideals?)

Collaboration Policy

With each week's homework, you must turn in a one paragraph description of all the resources you used on that homework. You must mention any person you talked to about the problems, any book you looked at, any online resource (Wikipedia, Chegg,...) that you used. A sample paragraph is

> On this week's homework, I worked on the problem set collaboratively with Gauss and Grothendieck at The Redroom during happy hour. We found an Alex Jones video (http://youtube.blah.com) that gave a really clear explanation of Fermat's Last Theorem. We compared our solutions against a solution key that we found on the /commutativealgebra/ board of 4chan (http://blah.blah.edu). We also got really stuck on Problem 5, and so we went to Chegg.com and paid an online tutor ("Zariski") $50 to solve the problem for us.