Math 201: Problem Set #4

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Winter 2024

Due in 1 week.

Problem 1

This problem concerns the module $M = \mathbb{R}^3$ over the ring $R = \mathbb{R}$.

1. Exhibit an explicit bases for M and $M \otimes_R M$.
2. By the universal property, the cross product $M \times M \to M$ corresponds to an element of $\mathrm{Hom}_R(M \otimes_R M, M)$. In terms of your bases, write down this element.

Problem 2

Solve Exercise 11 in section 10.4 of Dummit and Foote.

Problem 3

Solve Exercise 24 in section 10.4 of Dummit and Foote.

Problem 4

1. If R is a field and M a finitely generated R-module, then prove that $N \otimes_R \mathrm{Hom}(M, R)$ is isomorphic to $\mathrm{Hom}(M, N)$. Here $\mathrm{Hom}(M, R)$ and $\mathrm{Hom}(M, N)$ are given module structure by pointwise operations. (This isomorphism is frequently used in differential geometry.)
2. Are the two modules isomorphic if we drop the hypothesis that M is finitely generated?

Collaboration Policy

With each week's homework, you must turn in a one paragraph description of all the resources you used on that homework. You must mention any person you talked to about the problems, any book you looked at, any online resource (Wikipedia, Chegg,...) that you used. A sample paragraph is

> On this week's homework, I worked on the problem set collaboratively with Gauss and Grothendieck at The Redroom during happy hour. We found an Alex Jones video (http://youtube.blah.com) that gave a really clear explanation of Fermat's Last Theorem. We compared our solutions against a solution key that we found on the /commutativealgebra/ board of 4chan (http://blah.blah.edu). We also got really stuck on Problem 5, and so we went to Chegg.com and paid an online tutor ("Zariski") $50 to solve the problem for us.