Math 201: Problem Set #7

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Winter 2024

Due in 1 week.

Problem 1

Let M be the quotient of $\mathbb{Z}^3$ by the submodule generated by $(-4, -6, 7)$, $(2, 2, 4)$, and $(6, 6, 15)$. Describe M in terms of the classification of finitely generated $\mathbb{Z}$-modules. Do not use a computer.

Problem 2

Let $f = 11x - 181y + 139z$. Let M equal the vector space of homogeneous degree 2 polynomials in variables x, y, z with integer coefficients. Let N be the submodule generated by fx, fy, fz. Describe the quotient module M/N in terms of the classification of finitely generated $\mathbb{Z}$-modules. Use a computer or whatever other resources you want.

Problem 3

Solve Exercise 20 in section 12.1 of Dummit and Foote.

Problem 4

Let $A\colon \mathbb{Z}^2 \to \mathbb{Z}^2$ be a $\mathbb{Z}$-module homomorphism defined by multiplication by a integer matrix that we also denote by A.

1. Prove that A is injective if and only if, considered as a real matrix, it has rank 2.
2. Prove that A is surjective if and only if the determinant of A is ± 1.
3. Exhibit an example of an A that has rank 2 but is not surjective.

Collaboration Policy

With each week's homework, you must turn in a one paragraph description of all the resources you used on that homework. You must mention any person you talked to about the problems, any book you looked at, any online resource (Wikipedia, Chegg,...) that you used. A sample paragraph is

> On this week's homework, I worked on the problem set collaboratively with Gauss and Grothendieck at The Redroom during happy hour. We found an Alex Jones video (http://youtube.blah.com) that gave a really clear explanation of Fermat's Last Theorem. We compared our solutions against a solution key that we found on the /commutativealgebra/ board of 4chan (http://blah.blah.edu). We also got really stuck on Problem 5, and so we went to Chegg.com and paid an online tutor ("Zariski") $50 to solve the problem for us.