

# Math 201: Problem Set #9

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Winter 2024

Due in 1 week.

## Problem 1

1. Compute the rational canonical form of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{pmatrix}$$

over the field  $\mathbb{Z}/2$ .

2. Prove that  $A$  is not similar to a Jordan matrix over  $\mathbb{Z}/2$ .

## Problem 2

The set of similarity classes of elements of  $\text{Mat}_3(\mathbb{Z}/2)$  is finite. Explicitly describe this set by writing down one matrix for each similarity class. How many elements does this set have?

## Problem 3

Consider the matrix

$$A = \begin{pmatrix} 1 & -2 & -1 & 0 \\ 1 & 0 & -3 & 0 \\ -1 & -2 & 1 & 0 \\ 1 & 2 & 1 & 2 \end{pmatrix}.$$

Thinking of  $A$  as an element of  $\text{Mat}_4(\mathbb{C})$ , compute (1) the rational canonical form and (2) the Jordan canonical form. Do not use a computer algebra package.

**Problem 4**

Let  $p(x) = x^{10} - x^9 + x^8 - x^7 + x^6 - x^5 + x^4 - x^3 + x^2 - x + 1$ . As a polynomial in  $x$  and  $y$ ,  $p(x) - p(y)$  is divisible by  $x - y$ . Write

$$p(x) - p(y) = (x - y) \cdot \left( \sum a_{i,j} x^i y^j \right).$$

Compute both the rational canonical form and the Jordan canonical form of  $(a_{i,j})$ . (This is a big matrix, so you should figure out how to do this using a computer algebra package.)

**Problem 5**

Pick an exercise in Chapter 11 that looks interesting to you and then solve it.

**Collaboration Policy**

With each week's homework, you must turn in a one paragraph description of all the resources you used on that homework. You must mention any person you talked to about the problems, any book you looked at, any online resource (Wikipedia, Chegg,...) that you used. A sample paragraph is

On this week's homework, I worked on the problem set collaboratively with Gauss and Grothendieck at The Redroom during happy hour. We found an Alex Jones video (<http://youtube.blah.com>) that gave a really clear explanation of Fermat's Last Theorem. We got really stuck on Problem 5, and so we went to Chegg.com and paid an online tutor ("Zariski") \$50 to solve the problem for us. He said the problem was too hard for him. So I logged into my TruthSocial account (@CobraTatesThesis) and posted the question with @realDonaldTrump tagged. He responded with a tremendous, really fantastic solution to the problem, which by the way, Biden can't solve. At this point, it was midnight and I still had four more problems to go, so I just gave the questions to ChatGPT and cut-and-pasted the answers.