Math 194: Problem Set #3

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Due in 1 week.

Problem 1

Pick the topic for your final essay and describe one source that you will read to learn about the topic.

Problem 2

Identify problems with the use of mathematics in the following excepts of text, and rewrite them to fix the problems

1. Let $f(t)$ be Gryffindor's score t minutes into a game against Slytherin. f is globberfluxible at $t = 3$.
2. The function $z^2 + 1$ is even.
3. On a compact space every real-valued continuous function f is bounded.
4. If $0 \leq \lim_{n\to\infty} a_n^{1/n} = \rho \leq 1$, then $\lim_{n\to\infty} a_n = 0$.
5. The union of a sequence of measurable sets is measurable.
6. For invertible X, X^* is also invertible.
7. If $x > 0$, then Euler proved in 1756 that ....
8. Assume $x = 3$. Therefore $2x = 6$.
9. And when $x = -1$ instead, we can see that. . .
10. Let the angles of the triangle be δ, a_1, and t.
11. Let the number of elephants in the zoo in year n with $n \in [0, \infty)$ be $e(n)$. Suppose the growth rate is $\frac{de}{dn} = 2.5$.

12. Let's derive the formula

$$1 + r + \cdots + r^n = \frac{1 - r^{n+1}}{1 - r}, r \neq 1.$$

Define the LHS of the above equation to be $S_{r,n} = 1 + r + \cdots + r^n$. We compute

$$rS_{r,n} - S_{r,n} = r^{n+1} - 1.$$

If $r \neq 1$, divide both sides by $r - 1$ to obtain

$$S_{r,n} = \frac{r^{n+1} - 1}{r - 1}$$

which is equivalent to the first equation.

13. Let P be the escaped wombat population (in thousands) t years after 1990 and suppose that

$$P = 0.5(1.12)^t.$$

The wombat population in 1992 is approximately 672. We can see this by setting $t = 2$ and observing that

$$P = 0.5(1.12)^2 = 0.6272 \text{ thousand wombats.}$$

If we want to predict when the wombat population will reach 2000, we set $P = 2$ and solve for t using logarithms.

$$2 = 0.5(1.12)^t log2 = log0.5 + tlog1.12t = \frac{\log(2) - \log(0.5)}{\log(1.12)} \approx 12.23 \text{ years.}$$

The wombat population will reach 2000 in the year 2002.

14. (Proof that the functions e^x and e^{2x} are linearly independent over the reals.) Assume on the contrary that e^x and e^{2x} are linearly dependent. Then there exist constants c_1 and c_2, not both zero, such that

$$c_1 e^x + c_2 e^{2x} = 0 \text{ for all } x$$

Differentiating, we get $c_2 e^x = 0$, so $c_2 = 0$. But then $c_1 = 0$. Thus,

$$c_1 = c_2 = 0$$

This contradicts the fact that c_1 and c_2 are not both zero; therefore we must reject the assumption that the functions are linearly dependent. Consequently, they are linearly independent.

Collaboration Policy

With each week's homework, you must turn in a one paragraph description of all the resources you used on that homework. You must mention any person you talked to about the problems, any book you looked at, any online resource (Wikipedia, Chegg,...) that you used. A sample paragraph is

> On this week's homework, I worked on the problem set collaboratively with Gauss and Grothendieck at The Redroom during happy hour. We found an Alex Jones video (http://youtube.blah.com) that gave a really clear explanation of Fermat's Last Theorem. We compared our solutions against a solution key that we found on the /commutativealgebra/ board of 4chan (http://blah.blah.edu). We also got really stuck on Problem 5, and so we went to Chegg.com and paid an online tutor ("Zariski") $50 to solve the problem for us.