Robinson's Shorter Course.

THE

COMPLETE ALGEBRA,

DESIGNED FOR USE IN

SCHOOLS, ACADEMIES, AND COLLEGES.

BY

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hence the negative root is between -2 and -3. To separate the two roots which lie between 2 and 3 we must substitute for x some number or numbers lying between 2 and 3. When $x = 2\frac{1}{2}$ the signs of the functions are $-\ -\ \pm\ +$. Here we have only one variation whether we consider the vanishing function $f_1(x)$ to be positive or negative; hence one of the positive roots lies between 2 and $2\frac{1}{2}$, and the other between $2\frac{1}{2}$ and 3.

2. $x^3 - 3x^2 - 12x + 24 = 0$.

Ans. Three; one between 1 and 2, one between 4 and 5, and one between -3 and -4.

3. $x^3 + 6x^2 + 10x - 1 = 0$.

4. $x^3 - 6x^2 + 8x + 40 = 0$.

5. $x^4 + 4 = 0$.

6. $x^5 - 2x^4 + 3x^3 - 7x^2 + 8x - 3 = 0$.

7. $x^7 - 9x^5 + 6x^4 + 15x^3 - 12x^2 - 7x + 6 = 0$.

8. $x^4 + x^3 - x^2 - 2x + 4 = 0$.

HORNER'S METHOD OF APPROXIMATION.

615. Let it be required to find a root of the equation

$$x^n + Ax^{n-1} + Bx^{n-2} + \dots + Kx + L = 0 \quad . \quad . \quad . \quad (1).$$

Suppose a to be the integral part of the root required, and $r, s, t, \dots,$ taken in order, to be the digits of the fractional part.

Let a be found by trial (**608**) or by Sturm's Theorem; then find an equation whose roots shall be less by a than those of (1) (**598**).

Let $y^n + A'y^{n-1} + B'y^{n-2} + \dots + K'y + L' = 0 \dots (2)$ be that equation.

In this equation the value of y is less than 1; hence the terms containing the higher powers of y are comparatively small; neglecting these, we have, approximately,

$$K'y + L' = 0, \quad \text{whence} \quad y = -\frac{L'}{K'}.$$

The first figure in the value of y is r.

Now find an equation whose roots shall be less by r than those of (2). Let $z^n + A''z^{n-1} + B''z^{n-2} + \dots + K''z + L'' = 0 \dots (3)$ be that equation.

In this equation the value of z is less than .1; hence, we have, approximately, $K''z + L'' = 0$; whence $z = -\frac{L''}{K''}$. This process may be continued to any desired extent, and we shall have finally $x = a + r + s + t + \ldots.$

RULE.

I. *Find the integral part of the root by Sturm's Theorem or otherwise.*

II. *Find an equation whose roots shall be less than those of the given equation by the integral part of the required root.*

III. *Divide the independent term of the transformed equation by the coefficient of the adjacent term, change the sign of the quotient and write the first figure of the result as the first figure of the fractional part of the root.*

IV. *Find an equation whose roots shall be less than those of the second equation by the first figure in the fractional part of the required root.*

V. *Divide the independent term of this transformed equation by the coefficient of the adjacent term, change the sign of the quotient, and write the first figure of the result as the second figure of the fractional part of the required root.*

VI. *Continue this process until the root is obtained to the required degree of accuracy.*

SCH. 1.—To obtain the negative roots it is best to change the signs of the alternate terms of the given equation, and then find the *positive* roots of the result; changing the signs of these, we obtain the negative roots required.

SCH. 2.—If a trial figure of the root, obtained by any division, causes the two last terms of the succeeding equation to have the same sign, that figure is not the correct one and must be changed.

SCH. 3.—If K' should reduce to zero in the operation, then we should have, approximately, $J'y^2 + L' = 0$; whence $y = \sqrt{-\frac{L'}{J'}}$.

EXAMPLES.

1. Find one root of the equation $x^3 - 2x^2 - 20x - 40 = 0$.

By Sturm's Theorem we find that this equation has only one real root, and that the integral part of this root is 6. We now find two figures of the fractional part as follows:

1	-2	-20	-40	6.23
	$+6$	$+24$	$+24$	
	$+4$	$+4$	$-16^{(1)}$	
	$+6$	$+60$		
	$+10$	$+64^{(1)}$		
	$+6$			
	$+16^{(1)}$			
$1^{(1)}$	$+16^{(1)}$	$+64^{(1)}$	$-16^{(1)}$	
	$+0.2$	$+3.24$	$+13.448$	
	$+16.2$	$+67.24$	$-2.552^{(2)}$	
	$+0.2$	$+3.28$		
	$+16.4$	$+70.52^{(2)}$		
	$+0.2$			
	$+16.2^{(2)}$			
$1^{(2)}$	$+16.6^{(2)}$	$+70.52^{(2)}$	$-2.552^{(2)}$	

We find the coefficients of an equation whose roots are less by 6 than those of the given equation, using the method explained in Art. **598.** These coefficients are 1, 16, 64, and -16, marked (1) in the operation. Dividing 16 by 64, we obtain .2, which is the second figure of the root. We next find the coefficients of an equation whose roots are less by .2 than those of the second equation. These coefficients are marked (2) in the operation. Dividing 2.552 by 70.52, we obtain .03, which is the third figure of the root. This process may be continued until the root is obtained to any required degree of accuracy.

2. Find one root of the equation $x^4 + x^3 - 30x^2 - 20x - 20 = 0$.

By Sturm's Theorem, we find the integral parts of the two real roots to be 5 and -5. Changing the signs of the alternate terms of the equation, we find the fractional part of the negative root as follows:

1	-1	-30	$+20$	-20	5.73
	$+5$	$+20$	-50	-150	
	$+4$	-10	-30	$-170^{(1)}$	
	$+5$	$+45$	$+175$		
	$+9$	$+35$	$+145^{(1)}$		
	$+5$	$+70$			
	$+14$	$+105^{(1)}$			
	$+5$				
	$19^{(1)}$				
$1^{(1)}$	$+19^{(1)}$	$+105^{(1)}$	$+145^{(1)}$	$-170^{(1)}$	
	$+0.7$	$+13.79$	$+83.153$	$+159.7071$	
	$+19.7$	$+118.79$	$+228.153$	$-10.2929^{(2)}$	
	$+.7$	$+14.28$	$+93.149$		
	$+20.4$	$+133.07$	$+321.302^{(2)}$		
	$+.7$	$+14.77$			
	$+21.1$	$+147.84^{(2)}$			
	$+.7$				
	$21.8^{(2)}$				
$1^{(2)}$	$+21.8^{(2)}$	$+147.84^{(2)}$	$+321.302^{(2)}$	$-10.2929^{(2)}$	

Hence the negative root of the given equation is $-5.73+$.

Find the real roots of the following equations:

3. $x^3+2x^2-23x-70=0$. *Ans.* 5.1345.

4. $x^3-x^2+70x-300=0$. *Ans.* 3.7387.

5. $x^3+x^2-500=0$. *Ans.* 7.6172.

6. $x^3-x^2-40x+108=0$. *Ans.* 3.3792, 4.5875, −6.9667.

7. $x^3-4x^2-24x+48=0$. *Ans.* 1.7191, 6.5461, −4.2652.

8. $x^4+x^3+x^2-x-500=0$. *Ans.* 4.4604, −4.9296.

9. $x^4-9x^3-11x^2-20x+4=0$. *Ans.* .1796, 10.2586.