Math 23B: Problem Set #1

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Spring 2021

Due at the start of class on Monday April 4, 2022.

Problem 1

Write 1 paragraph explaining (1) your background in math and (2) your motivation for taking the course.

Problem 2

Read the syllabus. Do you have any questions? Did you catch any typos. If you catch a typo, let me know, and you will get extra credit that allows you to replace one homework grade with full credit.

Problem 3

Complete the practice problems on Math 23A material on pages the following.

Problem 4

Get the name and contact information for at least one other student in the class so that you have someone to contact if something comes up.

Collaboration Policy

With each week's homework, you must turn in a one paragraph description of all the resources you used on that homework. You must mention any person you talked to about the problems, any book you looked at, any online resource (Wikipedia, Chegg,...) that you used. A sample paragraph is

> On this week's homework, I worked on the problem set collaboratively with Gauss and Grothendieck at The Redroom during happy hour. We found an Alex Jones video (http://youtube.blah.com) that gave a really clear explanation of Fermat's Last Theorem. We compared our solutions

against a solution key that we found on the /commutativealgebra/ board of 4chan (http://blah.blah.edu). We also got really stuck on Problem 5, and so we went to Chegg.com and paid an online tutor ("Zariski") $50 to solve the problem for us.

1. For the vectors $a = \langle 5, 12 \rangle$ and $b = -3\mathbf{i} + -6\mathbf{j}$, compute the following quantities.

   (a) (10 points) $a + b$.

   (b) (5 points) $||a||$.

   (c) (5 points) $||a - b||$.

2. Compute the following.

   (a) (10 points) The dot product $a \cdot b$ for $a = (0, 4, -3)$ and $b = (2, 4, 6)$.

   (b) (5 points) Compute $a \times b$ for $a = \langle 1, 1, -1 \rangle$ and $b = \langle 2, 4, 6 \rangle$.

   (c) (5 points) Compute $(a \times b) \cdot a$ for $a = \langle 1, -1, -1 \rangle$ and $b = (1/2, 1, 1/2)$.

3. (20 points) Where does the line through $(1,0,1)$ and $(4,-2,2)$ intersect the plane $x+y+z=6$?

4. (a) (10 points) Do the following lines intersect:

$$L_1\colon x = 2 + t, y = 2 + 3t, z = 3 + t,$$
$$L_2\colon x = 2 + t, y = 3 + 4t, z = 4 + 2t?$$

Circle your answer:

A. They do intersect.

B. They do not intersect.

If the lines intersect, find the point of intersection.

(b) (10 points) Do the following lines intersect:

$$L_1\colon x = 1 + 7t, y = 3 + t, z = 5 - 3t,$$
$$L_2\colon x = 4 - t, y = 6, z = 7 + 2t?$$

Circle your answer:

A. They do intersect.

B. They do not intersect.

If the lines intersect, find the point of intersection.

5. For each of the following problems, circle the answer you think is correct. You do NOT need to justify your answer. In each part, $h(x, y, z) = \ln(x^2 + y^2 - 1) + y + 6z$ and P_0 is the point $(1, 1, 0)$.

   (a) (5 points) What is the directional derivative of h at P_0 in the direction of $\mathbf{u} = \langle -6, -1, 7 \rangle$.

   A. 0

   B. 1

   C. 27

   D. -84

   (b) (5 points) In which direction is h increasing the most rapidly?

   A. $\langle 0, -4, 1 \rangle / ||\langle 0, -4, 1 \rangle||$

   B. $\langle 2, 3, 6 \rangle / ||\langle 2, 3, 6 \rangle||$

   C. $\langle -6, -1, 7 \rangle / ||\langle -6, -1, 7 \rangle||$

   D. $\langle 6, 1, -7 \rangle / ||\langle 6, 1, -7 \rangle||$

   E. $\langle 0, -1, 0 \rangle$

6. For each of the following problems, circle the answer you think is correct. You do NOT need to justify your answer. This question concerns the surface defined by the equation $2z - x^2 = 0$ and the point $P_0 = (2, 0, 2)$.

   (a) (5 points) What is an equation for the tangent plane?

   A. $x + y + z = 3$

   B. $2x + 2y + z = 4$

   C. $3x - 4y + 5 = 0$

   D. $z = 0$

   E. $2x - z - 2 = 0$

   (b) (5 points) What is an equation for the normal line?

   A. $x = t, y = 0, z = 0$

   B. $x = 2 - 4t, y = 0, z = 2 + 2t$

   C. $x = t, y = t, z = -t$

   D. $x = 2t, y = 1 + 2t, z = 2 + t$

   E. $x = 1 + 2t, y = 1 + 2t, z = 1 + 2t$

7. (10 points) Find all the local maximum and minimum values and saddle point(s) of the function

$$f(x, y) = x^3y + 12x^2 - 8y.$$

Clearly indicate which points are which by writing **local max**, **local min**, or **saddle point**.

8. (10 points) Find three positive numbers whose sum is 12 and the sum of whose squares is as small as possible.