MATH 181 NOTES: FEBRUARY 4, 2024

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Over the past two classes, we have taken a look at Archimedes's text *The Sand Reckoner*. That text concerned large numbers (i.e. reckoning with numbers like number of grains of sand it would take to fill the universe). In reading the work in translation, we lost some important information, most notably the manner in which numbers are written. The Arabic numerals that we use today $(1, 2, 3, 4, ...)$ only came into use hundreds of years after the time of Archimedes. In this lecture, we will take a look at how the ancient Greek wrote numbers.

In fact, there were three or four different commonly used systems for writing numbers. (Recall that ancient Greece lasted for a thousand years and spanned a geographic area much larger than that of the modern nation of Greece.) The system was commonly used by people during Archimedes' time was the last system that was developed. The numerals are sometimes called the "Ionian" or "Alexandrian" numerals. The origins of this numerical system are unclear. The earliest examples date to circa 600 BC, although widespread only begins in 200 BC. The earliest examples are inscriptions on vases that were made for inventory purposes. Figure 1 (taken from Chrisomalis's article "The Egyptian origin of the Greek alphabetic numerals") is an inscription that dates to approximately 500 BC. The Greek letters κθ represented the number 29. The same number is written in a different numerical system in the upper-left corner (the vertical and horizontal lines).

How did this numerical system develop? An early theory held that it was an adopted from the system used by "Semetic people" (people in the Middle East such as Arabs, Jews, and Phoenicians), but most scholars have discounted this the nineteenth century. One viable theory is that the numerical system is an original Greek invention, while another holds that it was a modified form of a numerical system used by the Egyptians (the demotic numerals).

Figure 2. Inscription from the base of Louvre F211 (attributed to the Leagros Group, painter S, 525 – 500 BC). 2:1 scale. After Alan W. Johnston, Trademarks on Greek Vases, Figure 11d.

FIGURE 1. A Greek vase showing numbers

The basic idea behind the Greek numerical system is simple. Each Greek alphabetical character stand for a different numbers, and characters coming later in the alphabet have larger values. The exact values are:

I. ONES	II. TENS	III. HUNDREDS
$\alpha = 1$	$\iota = 10$	$\rho = 100$
$\beta = 2$	$\kappa = 20$	$\sigma = 200$
$\gamma = 3$	$\lambda = 30$	$\tau = 300$
$\delta = 4$	$\mu = 40$	$\upsilon = 400$
$\epsilon = 5$	$\nu = 50$	$\phi = 500$
$? = 6$	$\xi = 60$	$\chi = 600$
$\zeta = 7$	$o = 70$	$\psi = 700$
$\eta = 8$	$\pi = 80$	$\omega = 800$
$\theta = 9$	$? = 90$	$? = 900$

The three numbers marked with questions marks were denoted by symbols that were not alphabetic characters. The number six is represented by what is called the digamma symbol, $\digamma = 6$. The symbols denoting 90 and 900 are respectively the koppa and the sampi. I can't figure out how to type the symbols, but they are displayed in Figures 2 and 3. Rather, they usually they look like that. The shapes of all Greek characters evolved over time, but especially the digamma, koppa, and sampi, evolved over time. The Katz textbook shows some other ways these symbols can appear.

ϙ

FIGURE 2. The symbol for 90, the koppi

Ͳ

FIGURE 3. The symbol for 900, the sampi

. . σεις βοῦν χοινικιαῖον ἀνυπε[ρθέ]-
τως. νν Μουνιχιῶνος $\overline{\beta}$ ἀπιόντος Ἡρ̣[α]-
κλεῖ καὶ θειῷ ἀλέκτορας $\overline{\beta}$, πόπαν[α]
χοίνικος δωδεκόμφαλα ὀρθόνφαλ[α]
ἀνυπερθέτως.

FIGURE 4. Part of a sacrificial calendar

Numbers were usually written by concatenating characters from left to right (i.e. the characters are listed in descending order). For example the number 54 is written νδ. However, observe that, unlike our Arabic numbers the position of a Greek numerical character is irrelevant. For numbers between 11 and 19, the order of the characters is sometimes reversed, so 12 is written βι rather than ιβ.

One feature of the Greek numerical system is that the same character is used to denote both a number and a Greek letter, so a reader needs to use the context to figure out whether or not they are reading a number. To avoid confusion, numbers are often overlined. Thus β denotes the second letter in the Greek alphabet, but $\overline{\beta}$ denotes the number 2. Let's take at some examples. Figure 4 shows a transcription of an inscription found in a church in Athens, Greece that is currently held by the Ashmolean Museum at Oxford University (catalogued as AIUK 11). The inscription is believed to be a sacrificial calendar that was written circa 2 AD. Observe that the number $\overline{\beta}$ appears twice. Here is one translation of the the text relevant part of the text:

> Mounichion: On the 2nd to last day of the month, for Herakles and his Uncle, 2 roosters, round cakes containing a choinix with twelve bosses, straight-bossed, (sacrifice) immediately

Figure 5 shows another portion of the inscription. Two numbers appears: $\overline{\gamma\iota}$ and $\overline{\zeta\iota}$. These numbers are 13 and 17 respectively. Observe that the symbols have been written right-to-left.

Another example is displayed in Figure 6. That text is a description of how to compute square roots. It is taken from a mathematics book that was written by Theon of Alexandria

Βοηδρομιῶνος γ̅ι̅ Νέφθυϊ καὶ Ὀσίριδ[ι]
ἀλεκτρυόνα καρπώσεις σπείρων πυρ[οὺς]
καὶ κρειθάς, σπένδων μελίκρατον. ζ̅ι̅ Δήμ[η]-
τρι Κόρῃ δ̣έ̣λφακα ἀνυπερθέτως. ηι τρυγ[η]-
τὸν Διονύσῳ καὶ τοῖς ἄλλοις θ̣ε̣οῖς ἀν[υ]π̣ερ[θ](έτως)·

FIGURE 5. Another part of a sacrificial calendar

ἂν δοθέντος χωρίου τινὸς τετραγώνου μὴ ἔχοντος πλευρὰν μήκει ῥητὴν τὴν σύνεγγυς αὐτοῦ τετραγωνικὴν πλευρὰν ἐπιλογισώμεθα. καὶ ἔστιν τὸ τοιοῦτον δῆλον ἐπὶ ῥητὴν ἔχοντος πλευράν, ἐκ τοῦ δʹ θεωρήματος τοῦ βʹ βιβλίου τῶν Στοιχείων, οὗ ἡ πρότασίς ἐστιν τοιαύτη· ἐὰν εὐθεῖα γραμμὴ τμηθῇ ὡς ἔτυχεν, τὸ ἀπὸ τῆς ὅλης τετράγωνον ἴσον ἐστὶν τοῖς τε ἀπὸ τῶν τμημάτων τετραγώνοις καὶ τῷ δὶς ὑπὸ τῶν τμημάτων περιεχομένῳ ὀρθογωνίῳ. ἐὰν γὰρ ἔχοντες δοθέντα ἀριθμὸν τετράγωνον ὡς τὸν ρ̅μ̅δ̅, ῥητὴν ἔχοντα πλευρὰν ὡς τὴν ΑΒ εὐθεῖαν, καὶ λαβόντες αὐτοῦ ἐλάσσονα τετράγωνον τὸν ρ̅, οὗ ἐστιν πλευρὰ ι̅, καὶ ὑποθέμενοι τὴν ΑΓ ι̅, διπλασιάσαντες αὐτὴν [καὶ][1] διὰ τὸ δὶς ὑπὸ τῶν ΑΓ, ΓΒ, ⟨παρὰ⟩[2] τὰ γενόμενα κ̅ παραβάλωμεν [παρὰ][3] τὰ λοιπὰ μ̅δ̅, τῶν ὑπολειπομένων δ̅ ἔσται τὸ ἀπὸ τῆς ΓΒ, αὕτη δὲ μήκει β̅· ἦν δὲ καὶ ἡ ΑΓ ι̅· καὶ ὅλη ἄρα ἡ ΑΒ ἔσται μοιρῶν ι̅β̅, ὅπερ ἔδει δεῖξαι.

FIGURE 6. Text by Theon of Alexandria

that was written at some point in the AD 300's. The first number that appears is $\overline{\rho\mu\delta}$ which is 144.

(It would be natural to include some examples from Archimedes, but I could not find any text in the original Greek.)

Observe that this numerical system is different from both the modern Western numerical system. While both systems build numbers from a few basic characters, the position of the characters matters in our Western numerical system, while it is irrelevant in the Greek system. Å second difference is that characters cannot be repeated in the Greek system (e.g. $\overline{\kappa\kappa}$ is not a valid way to write 40). The Greek numerical system also has similar differences from Roman numerals.

One problem with the numerical system is that there is no way of writing down 1000, or any larger number. Beginning around 500 BC, the numerics were extended. Each number from α to θ can be modified by adding a slating mark, called a hasta, to the left that multiplies its value by 1000. I can't type a hasta well, but the character for 1000 looks something like $,\alpha$. An examples of this character appears in Figure 7. That text is a selection from the section of Aristotle's *Problems* where he explains how someone can count large numbers using their hands. The numbers that appear are $\overline{\gamma}$ (or 3) and $,\overline{\gamma}$ (or 3000).

δὲ λοιπῶν δύο ἐκτεταμένων, τοῦ λιχανοῦ λέγω καὶ τοῦ ἀντίχειρος, εἰσὶν ἅπερ κρατεῖς ἐν μὲν τῇ λαιᾷ, $\bar{\gamma}$, ἐν δὲ τῇ δεξιᾷ, $,\bar{\gamma}$.

FIGURE 7. Selection from Aristotle's *Problems*

A natural system would be to add a hasta to numbers above θ, so that $,\bar{\iota} = 10,000$, etc. However, this system was not in common use. Instead, the symbol M (for "myriad") was used to denote $10,000$. Numbers larger than this are hard to find, and different writers use different notation. One approach was to write $\overset{x}{M}$ for the product of$10,000$ and the value of x. For example, $\overset{\beta}{M}$ equals $20,000$.

There's certainly more to be said about numbers in ancient Greek. In terms of text, we could explore the words for numbers in ancient Greek. We could also explore how the ancient Greeks actually performed computations. These topics are for another time.

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