# Math 194: Possible topics for Senior Seminar Thesis

(This list is meant only for a suggestion. You can choose a topic of your choice, as long as there is a substantial mathematical content. There are a variety of online resources for math topics.)

- Symmetry groups of Platonic solids (5 regular solids have interesting symmetries. You can also discuss finite subgroups of three dimensional rotations and reflections, Poincare manifold, etc.)
- Infinitesimals and non-standard analysis (history of the use of infinitesimals by Cavalieri, Leibniz, Euler, etc, and a modern approach by Abraham Robinson using hyperreal numbers)
- Permutation groups and simplicity of A_n (alternating group) for n>4. (How this is related to unsolvability of algebraic equations of degree bigger than 4 by radicals. Discuss history including Galois and Abel. Also, how to solve degree 3 and 4 equations.)
- Various proofs of Fundamental Theorem of Algebra (Original Gauss' proof in his thesis and by others, modern proofs using various methods from algebra, analysis, topology, etc.)
- Gauss-Bonnet Theorem (for surfaces with or without boundaries)
- Poincare-Hopf Theorem for vector fields
- Homological algebra (chain complex, homology, cohomology, etc)
- Constructible polygons and Fermat primes (in particular, discovery of the constructability of heptadecagon by Gauss, and how to actually draw it using ruler and compass)
- Constructions with straightedge and compass, and use of other tools such as ruler with marked points and origami to overcome the difficulty (in relation to trisecting angles and doubling cubes)

- Problem of Apollonius (how to draw a circle tangent to given three circles. More generally, drawing circles tangent to three objects, where objects can be a point, a line, or a circle. Discuss history, many solution methods, by inversion, by Gergonne, etc. Generalization to Apollonian gasket, which is a fractal.)
- Three body problem and Lagrange points (discovery of classical solutions to the 3-body problem by Euler and Lagrange, and Lagrange points for Earth-Sun system)
- Quaternions, Cayley numbers (these are real 4-dimensional and 8-dimensional division algebras. History of discovery and their use in algebra, topology and geometry. Relations to Lie groups. Use of quaternions in computer graphics)
- Elliptic functions (Doubly periodic meromorphic functions on the complex plane, and its relation to elliptic integrals, lemniscatic integrals)
- Conformal mappings (Schwarz-Christoffel formulae mapping the unit disc to polygons. Riemann mapping Theorem)
- Sums of (two, three, four) squares (History and discussion of proofs)
- Continued fractions (discuss many uses of continued fractions, continued fractions of famous numbers, Pells equations, etc)
- p-adic numbers
- Isoperimetric problems
- Archimedes' achievements (method of indivisibles, calculation of pi, area of a sphere, volume of a ball, area of a region bounded by a parabola and a line,  Archimedes Palimpsest, engineering inventions, etc.)
- Knot theory (discuss various invariants of knots, Jones polynomials, relation to quantum field theory, etc.)

- Euler's calculation of zeta function values at even integers (Basel problem, original method of Euler and modern methods of calculation)
- Gamma functions (generalization of factorial function and its appearance in many contexts including integral transforms)
- Construction of continuous fractal space filling curves (methods by Peano, Hilbert, etc)
- Fractals (Mandelbrot sets, Julia sets, etc)
- The Brachistochrone (a famous classical problem posed by Bernouilli. History around this problem and solutions.)
- Cycloid (early method of calculating the area and its length, tautochrone curve, isochronous curve, involute)
- Conic sections (study by Apollonius, Dandelin spheres, numerous properties including light reflection, projective geometry, applications)
- Calculus of variations: Euler-Lagrange equations (These are common tools in physics. Discuss theory and some examples from mechanics, electromagnetism, etc.)
- The prime number Theorem
- Collatz conjecture (easy to state, hard to understand the phenomenon)
- Fermat's Last Theorem for exponent 3 (or 4)
- Fibonacci numbers (history and their numerous properties and its appearance in many contexts in nature)
- Hyperbolic geometry
- Representations of finite groups (properties of characters of representations)
- Transcendence of e
- Transcendence of pi
- Fixed point theory (Theorems by Bouwer, Lefschetz, etc)

- Euler’s formula for convex polyhedra and its generalization (V-E+F=2, and formula for general case)
- Theory of partitions of integers (many topics, Ramanujan’s congruences, etc)
- Riemann zeta function: Euler product, functional equation, Riemann hypothesis
- Kepler conjecture on sphere packing (history and modern computer solutions)
- Differential forms in vector calculus (how differential forms simplify vector calculus, Stokes Theorem)
- Special relativity
- General relativity (Discuss Einstein field equations describing gravity as curvature of space-time, and Schwarzschild solutions, black holes, etc)
- Heat equation/Fourier series (history and methods on how to understand heat propagation)
- Hypergeometric equation (regular singular points, solutions in complex domains, etc)
- Poincare conjecture (history and current state in all or some dimensions)
- RSA cryptosystem
- Category theory (categorical description of various familiar concepts such as products, coproducts, pull-backs, push-outs, limits, kernels, cokernels, etc)
- Algebraic and transcendental numbers, Liouville numbers
- Five Squarable Lunes (Discovery from ancient times to modern times)
- Splines and Bezier curves (simple ways to draw and control curves in computer graphics)

- Euler-Mascheroni constant ϒ (gamma) (This number appears in many different contexts)
- Euler-Maclaurin summation formula (difference of finite sum and integrals)
- Stirling's formula for n!
- Cantor type sets, Cantor functions
- Jordan curve theorem
- Weierstrass function (continuous nowhere differentiable functions, and other pathological but interesting functions)
- Golay code
- Banach Tarski Paradox (A single solid ball of radius 1 is partitioned into 5 pieces and they are reassembled into two solid balls both of radius 1. Its generalization)
- Fluid dynamics of sport balls (how different features of balls affect their behavior while in motion in the air)
- Baire Category Theorem, nowhere dense sets (Analogy with measure zero sets)
- Mathematics in the Islamic world (historical role Islamic world played in mathematics, in preservation, development, and dissemination)
- Bernoulli numbers and the sum of q-th power of integers from 1 to n.
- Axiom of Choice (overview) (its diverse appearance in many areas of mathematics, including Zorn's Lemma, Well-ordering theorem, Banach Tarski paradox, etc.)
- Pi (history of calculation of pi, modern approaches, infinite product formulae, transcendence, rapidly convergent series, Ramanujan series, numerous appearances of pi in mathematics)