Math 19A **Name (Print):** _____
Spring 2023
Practice Exam 2
June 12, 2023
Time Limit: Not applicable

This exam contains 8 pages (including this cover page) and 7 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Clearly indicate your final answer** by for example circling it.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	15	
2	15	
3	15	
4	15	
5	15	
6	15	
7	15	
Total:	105	

1. Find the derivative of the following functions. Simply your answer as much as possible.

   (a) (5 points) $f(x) = 5x^4 - 1/2x^5 + 1/x$.

   (b) (5 points) $f(x) = (1 + \cos(x))(x - \sin(x))$.

   (c) (5 points) $f(x) = (4 + 1/x)(2x - 1/x^2)$.

2. Consider the function whose graph is given below.

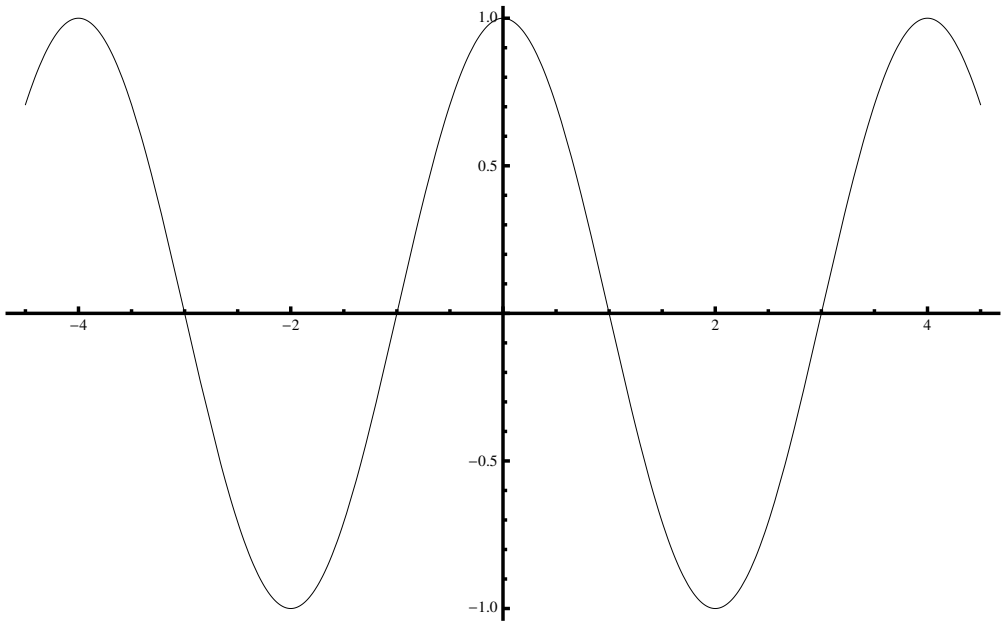

Figure 1: Graph of $y = f(x)$

For the following problems, you will need to estimate x values from the graph. Your numbers need to be within .5 of the actual value

(a) (3 points) For what values of x does the graph of $f(x)$ have a horizontal tangent line? If there are none, write "None".

(b) (3 points) Over what intervals does the graph of $f(x)$ have tangent lines with positive slope. If there are none, write "None".

(c) (3 points) Over what intervals does the graph of $f(x)$ have tangent lines with negative slope. If there are none, write "None".

(d) (3 points) For what values of x does the graph of $f(x)$ have an inflection point? If there are none, write "None".

(e) (3 points) Over what intervals is the graph of $f(x)$ concave upward. If there are none, write "None".

3. In each part, use the given equation to find $\frac{dy}{dx}$ in each of the following questions. Simplify your answer as much as possible.

   (a) (5 points) $xy^2 - x^2 + 4 = 0$.

   (b) (5 points) $(x^2 + y^2)^6 = x^3 - y^3$.

   (c) (5 points) $y = \frac{\sqrt{x^2+7}(x^3-5)^{1/3}\sin(x)}{x^4-7x+5}$. (Hint: Using a logarithm helps.)

4. (15 points) Let $f'(x) = (2 - x^2)/(x^2 + 2)^2$ be the first derivative of a continuous function $f(x)$. Find all critical points of f and determine whether each is a relative maximum, relative minimum, or neither. You must show enough work to justify your answers.

5. (15 points) If such a function exists, sketch in the space below the graph of a function $y = f(x)$ with the following properties:

   1. the domain of f is $[-5, 5]$;
   2. $f(0) = 5$, $f(2) = 0$;
   3. $f'(2) = 0$, $f''(3)$ does not exist;
   4. $f''(x) > 0$ for $x < 3$;
   5. $f''(x) < 0$ for $x < 3$;

   If no such function exists, explain why.

6. Evaluate each of the following limits.

(a) (4 points) $\lim_{x\to 1} \frac{2x-2}{\ln(x)}$

(b) (4 points) $\lim_{x\to 1} \frac{x^2+4}{x^2+1}$.

(c) (4 points) $\lim_{x\to 0} \frac{1}{e^x-1} - \frac{1}{x}$.

(d) (3 points) $\lim_{x\to\infty} x(e^{1/x} - 1)$.

7. (15 points) Consider the following problem:

   A student starts walking West down Gervais St from the State House at 4 ft/s. Five minutes later a professor starts walking North up Main Street from a point 500 ft North of the State House at 5 ft/s. At what rate are the people moving apart 15 minutes after the professor starts walking?

   Please show work and explain your reasoning.