Math 19A **Name (Print):** _____
Spring 2023
Practice Final
6/9/23
Time Limit: Class period

This exam contains 20 pages (including this cover page) and 18 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, or any calculator on this exam.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Clearly indicate your final answer** by for example circling it.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	15	
9	15	
10	15	
11	15	
12	15	
13	15	
14	15	
15	15	
16	15	
17	20	
18	15	
Total:	310	

1. The function $f(x)$ has range $[-1, 1]$ and satisfies $f(0) = 0$. Use the graph of $f(x)$ to answer the questions below. All numerical answers must be within .25 of the correct answer to receive full credit.

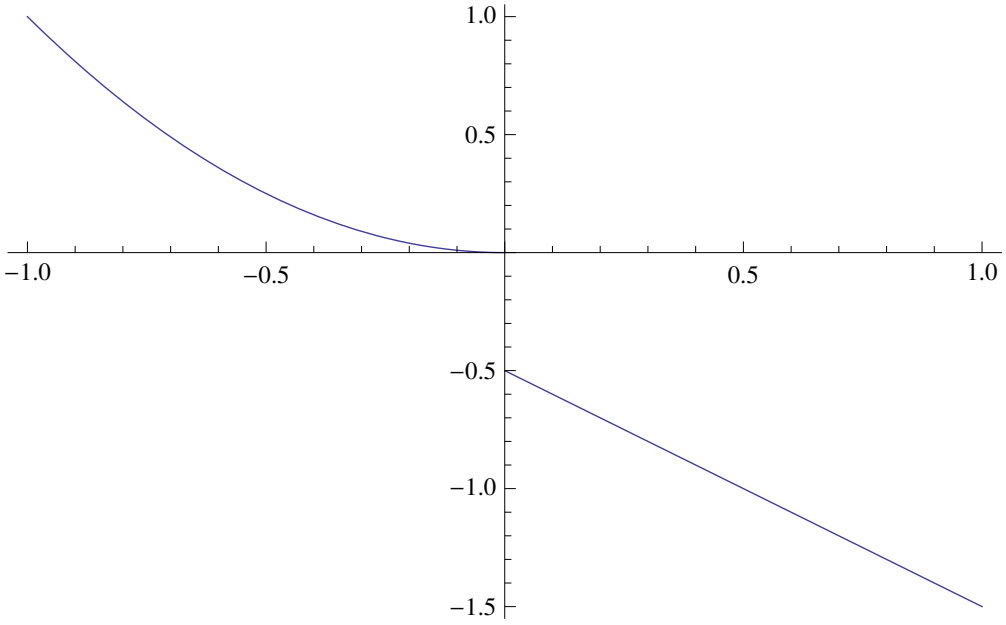

Figure 1: Graph of $y = f(x)$

(a) (5 points) What is $\lim_{x \to .5^-} f(x)$?

(b) (5 points) What is $\lim_{x \to .5^+} f(x)$?

(c) (5 points) What is $\lim_{x \to 0^-} f(x)$?

(d) (5 points) What is $\lim_{x \to 0^+} f(x)$?

2. Evaluate the following limits.

   (a) (10 points)

$$\lim_{x \to 1} \frac{x^2 + 3x - 4}{x - 1}$$

   (b) (10 points)

$$\lim_{x \to 4} \frac{\sin^2(\pi x)}{e^{x-4} + 3 - x}$$

3. (20 points) The radius of a circle is changing at the rate of $-2/\pi$ meters-per-second. At what rate is the circle's area changing when the radius is 10 meters.

Problems taken from Thomas's *Calculus*.

4. Consider the function $f(x) = 3x^2 - 4x^3$.

   (a) (10 points) Find the open intervals on which the function $f(x)$ is increasing and decreasing.

   (b) (10 points) Identify the function's local and absolute extreme values, if any, saying where they occur.

5. (20 points) Your iron works has contracted to design and build a 500 cubic feet, square-based, open-top, rectangular steel holding tank for a paper company. The tank is to be made by welding thin stainless steel plates together along their edges. As the production engineer, your job is to find dimensions for the base and height that will make the tank weight as little as possible. What dimensions do you tell the shop to use?

6. Find the derivatives of the following functions.

(a) (10 points)

$$y = \ln(\sin^2(x))$$

(b) (10 points)

$$y = \frac{\sqrt{t}}{1+\sqrt{t}}$$

7. Find the dy/dx using implicit differentiation.

   (a) (10 points)

$$5x^{4/5} + 10y^{6/5} = 15$$

   (b) (10 points)

$$y^2 = \frac{x}{x+1}$$

8. In this question $f(x) = -2x^3$ and $g(x) = 1/\sqrt[3]{x}$. Simplify your answer as much as possible. if the function is undefined, write *Undefined*.

   (a) (5 points) Find a formula for $f(x)/g(x)$.

   (b) (5 points) Find a formula for $f \circ g(x)$.

   (c) (5 points) Find a formula for $g \circ f(x)$.

9. (a) (5 points) What is the domain of $f(x) = x/(\sqrt[4]{x} + 1)$?

(b) (5 points) What is the domain of $g(x) = e^x/(x^2 - 3x + 2)$?

(c) (5 points) What is the domain of $h(x) = e^3/(e^{2x} + 1)$?

10. For the following functions, find a formula for the inverse function $f^{-1}(x)$ or explain why the inverse does not exist.

(a) (7 points) $f(x) = (5x - 2)/(x - 4)$ for all $x > 10$.

(b) (8 points) $g(x) = \ln(\frac{5x-3}{x})$ for $x > 3$.

11. (15 points) Use the space provided to sketch the graph of the function

$$f(x) = \begin{cases} x^2 & x \leq -4; \\ -\sqrt{16 - x^2} & -4 < x < 7; \\ 7 - x & x \geq 7. \end{cases}$$

12. (15 points) Draw a rough sketch of the following functions. Label your axes sufficiently well to show shifts and stretches.

(a) (7 points) $f(x) = -(x-3)^2 + 2$.

(b) (8 points) $f(x) = 1/(x+2) - 2$.

13. (a) (5 points) Express the following function as a rational function of x:

$$\ln((e^x)^2)/e^{5x})e^{\ln x}.$$

(b) (5 points) Find a formula for

$$\sin(\arcsin(4/5)) + \cos(\arcsin(4/5))$$

involving only rational numbers, power, and roots.

(c) (5 points) Express the following function as a rational function of x:

$$e^{\sin(\arcsin(\ln(x)))}.$$

Here $-1 \leq x \leq 1$.

14. Compute the following limits. If the limit does not exist, write *Does Not Exist*.

(a) (5 points) $\lim_{x\to\infty} \frac{99x^6+3x^{10}+2}{x+4x^{10}+1}$.

(b) (5 points) $\lim_{x\to\infty} \frac{x^3+1}{x^2+3}$.

(c) (5 points) $\lim_{x\to 0} \frac{2\sin(x)^2-2}{\sin(x)-1}$

15. Consider the function $f(x)$ whose graph is the following one.

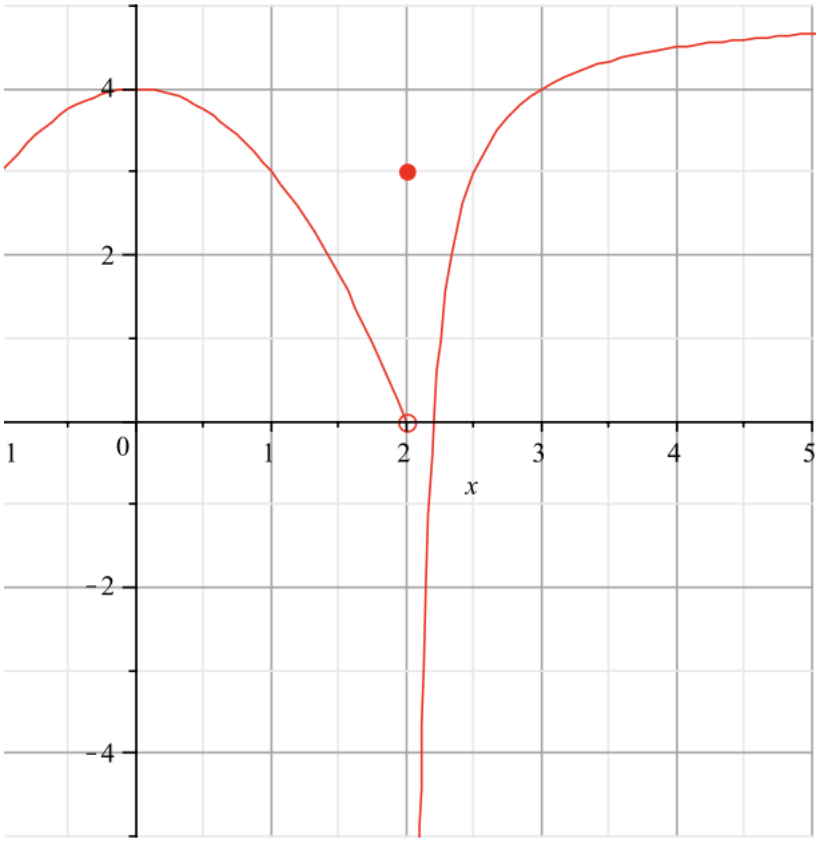

Figure 2: Graph of $y = f(x)$

Use the graph to answer the following questions.

(a) (5 points) What is $\lim_{x \to 0} f(x)$?

(b) (5 points) What is $\lim_{x \to 2^-} f(x)$?

(c) (3 points) What is $f(2)$?

(d) (2 points) Is $f(x)$ continuous at $x = 2$?

16. (15 points) Sketch the graph of a function f with all the following properties:

    1. the domain of f of $[-2, 3]$;
    2. $f(-2) = f(0) = f(3) = 0$;
    3. $\lim_{x\to 0} f(x) = 0$;
    4. $\lim_{x\to 2^+} f(x) = 1$;
    5. $\lim_{x\to 2^-} f(x) = 2$;
    6. f is not continuous at $x = -1$.

17. A car is driven from Columbia to Charleston. The driver does not take any rests or detours and obeys the speed limit. Let $P = f(t)$ be the distance in miles from Columbia after driving for t hours.

(a) (5 points) State the algebraic definition of the derivative of the function $f(x)$ at a.

(b) (5 points) What are the units of $f'(3)$?

(c) (5 points) Given the meaning of f, which of the following values of the derivative $f'(1)$ is most reasonable?

1. $f'(1) = 65$.
2. $f'(1) = 0.5$;
3. $f'(1) = -50$;
4. $f'(1) = -.4$.

(d) (5 points) Now suppose that $f(t) = 65t$. Use the definition of the derivative to find $f'(2)$.

18. (15 points) Use the definition of the derivative to compute $f'(a)$ for $f(x) = x^2 + 2x + 3$.